Option Discovery Algorithms

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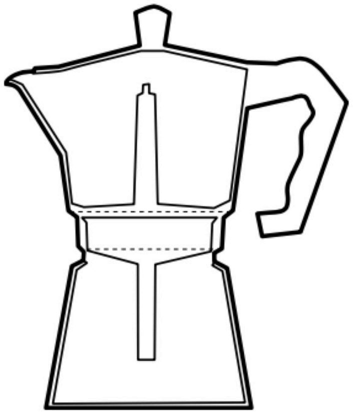
Overview

- Introduction
  - Temporal Abstraction in RL
  - Options
  - Semi-MDP
- Option Discovery Algorithms
  - policy gradient based methods
  - information-theoretic based
  - Variational options discovery

Temporal abstraction



High level steps

- Grind the beans, measure the right quantity of water, boil the water

Low level steps

- Wrist and arm movements while adding coffee to the filter, ...

Temporal abstraction in AI

A cornerstone of AI planning since the 1970’s:

- Fikes et al. (1972), Newell (1972, Kuipers (1979), Korf (1985), Laird (1986), Iba (1989), Drescher (1991) etc.

It has been shown to :

- Generate shorter plans
- Reduce the complexity of choosing actions
- Provide robustness against model misspecification
- Improve exploration by taking shortcuts in the environment

Temporal abstraction in RL

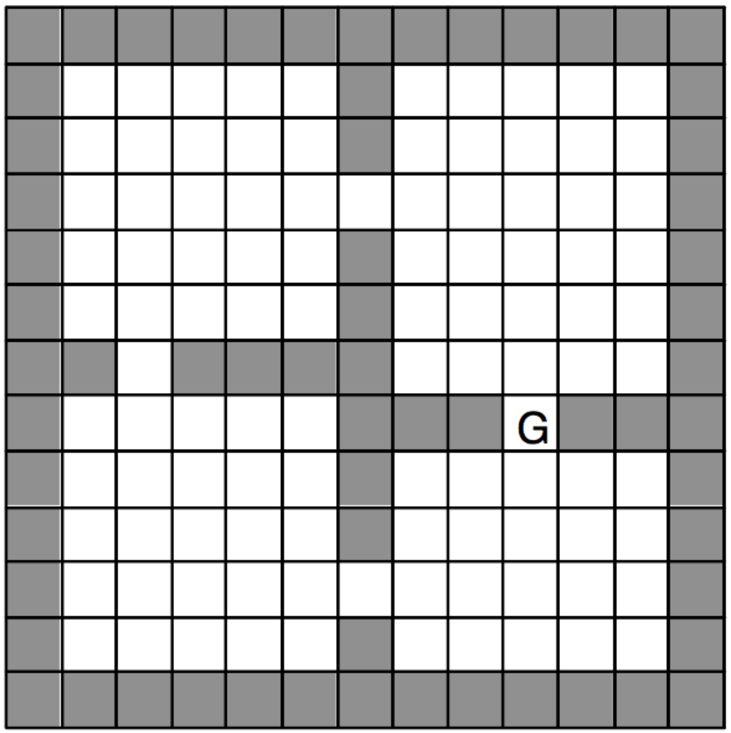
How can an agent represent stochastic, closed-loop, temporally-extended courses of action?

- HAMs (Parr & Russell 1998; Parr 1998)
- MAXQ (Dietterich 2000)
- **Options** (Sutton, Precup & Singh 1999; Precup 2000)

options - skills - macros - temporally abstract actions

(Sutton, McGovern, Dietterich, Barto, Precup, Singh, Parr…)

Example



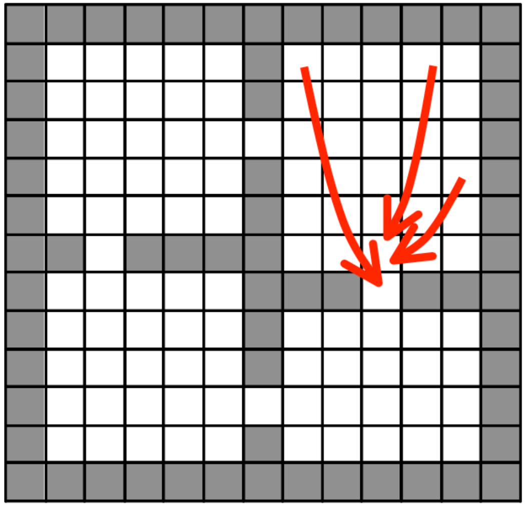
Actions
- North, East, South, West

Reward
- +1 for transitions into G
- 0 otherwise

$\gamma = 0.9$

Options

- A generalization of actions
- Starting from an initiation state, specify a way of choosing actions until termination
- Example: go-to-hallway



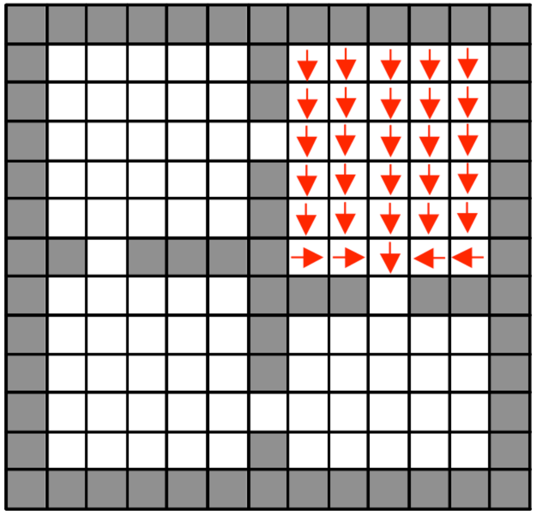
Markov Options

- An option can be represented as a triple:

$$o = < I, \pi, \beta >$$

- $I \subseteq S$ is the set of states in which *o* may be started
- $\pi: S \times A \rightarrow [0,1]$ is the policy followed during *o*
- $\beta: S \rightarrow [0,1]$ is the probability of terminating in each state

Example: go-to-hallway



One-Step Options

- A primitive action $a \in \cup_{s \in S} A_s$ of the base MDP is also an option, called a one-step option:
  - $I = \{s : a \in A_s\}$
  - $\pi(s, a) = 1, \forall_S \in I$
  - $\beta(s) = 1, \forall_S \in S$

Markov vs. Semi-Markov Options

- Markov option: policy and termination condition depend only on the current state
- Semi-Markov option: policy and termination condition may depend on the entire history of states, actions, and rewards since the initiation of the option
  - Options that terminate after a pre-specified number of time steps
  - Options that execute other options

Semi-Markov Options

- Let H be the set of possible **histories** (segments of experience terminates in s_τ, $\tau = t + k$)

$$H = < s_t, a_t, r_t, s_{t+1}, \dots, s_\tau >$$

- An semi-Markov option is represented as a triple:

$$o =< I, \pi, \beta >$$

  - $I \subseteq S$ is the set of states in which *o* may be started
  - $\pi: H \times A \rightarrow [0,1]$ is the policy followed during *o*
  - $\beta: H \rightarrow [0,1]$ is the probability of terminating in each state

Policy over Options

- Let μ be the policy over options. μ selects an option $o \in \ O_{s_t}$ according to probability distribution $\mu(s_t)$
$$\mu : S \times O \rightarrow [0,1]$$
- μ determines a conventional policy over actions, or *flat policy*, $\pi \ = \ flat(\mu)$.

Value functions for options

- Define $Q^{\mu}(s, o)$ the value of taking option $Q^{\mu}(s, o)$ in state s under policy μ, as

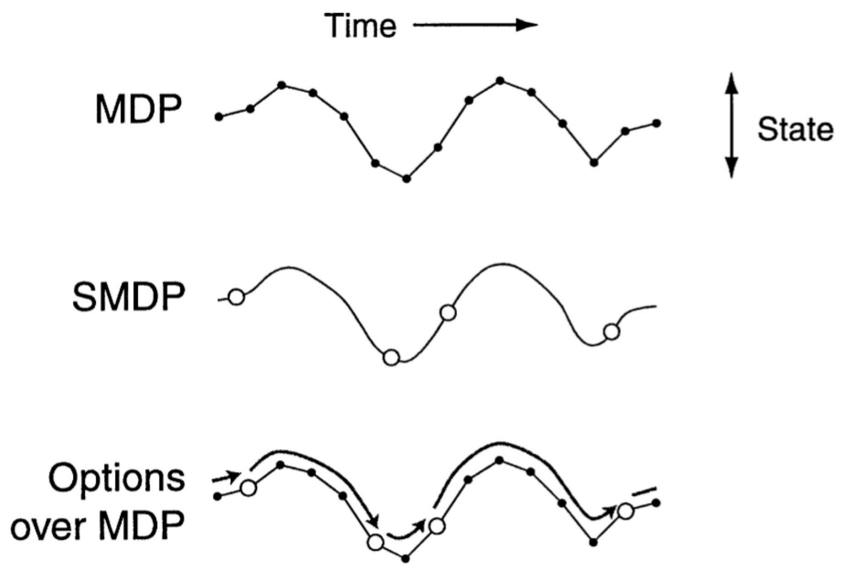
$$Q^{\mu}(s, o) \stackrel{\text{def}}{=} E\{r_t + \gamma r_{t+1} + \cdots \mid$$

$$o \text{ initiated in } s \text{ at time t}, \mu \text{ followed after termination}\}$$

$$Q^{*}(s, o) \stackrel{\text{def}}{=} max_{\mu \in \Pi(O)} Q^{\mu}(s, o)$$

- $\Pi(O)$ is the set of all policies selecting only from options in O

Options define a Semi-Markov Decision Process (SMDP)



- The state trajectory of an MDP is made up of discrete-time transitions and homogeneous discount.
- SMDP comprises larger, continuous-time transitions and discrete events and interval-dependent discount.
- **Options enable an MDP trajectory to be analyzed in either way.**

MDP + Options = SMDP

SMDPs

- The amount of time between one decision and the next is a random variable τ
- Transition probabilities $p(s', \tau | s, a)$
- Bellman equations

$$V^*(s) = \max_{o \in A_s} \left[R(s,a) + \sum_{s',\tau} \gamma^{\tau} P(s', \tau \mid s, a) V^*(s') \right]$$

$$Q^*(s,a) = R(s,a) + \sum_{s',\tau} \gamma^{\tau} P(s', \tau \mid s, a) \max_{o' \in A_{s'}} Q^*(s', a')$$

Option models

The reward of o:

- Let $\varepsilon(o, s, t)$ denote the event of o being initiated in state s at time t.

$$r_s^o = E\{r_t + \gamma r_{t+1} + \cdots + \gamma^{\tau-1} r_{t+\tau} | \varepsilon(o, s, t)\}$$

Transition probabilities:

- For all $s \in S$, $p(s', \tau)$ is the probability that the option terminates in s after τ steps.

$$p_{ss'}^o = \sum_{\tau=1}^{\infty} p(s', \tau)\gamma^\tau$$

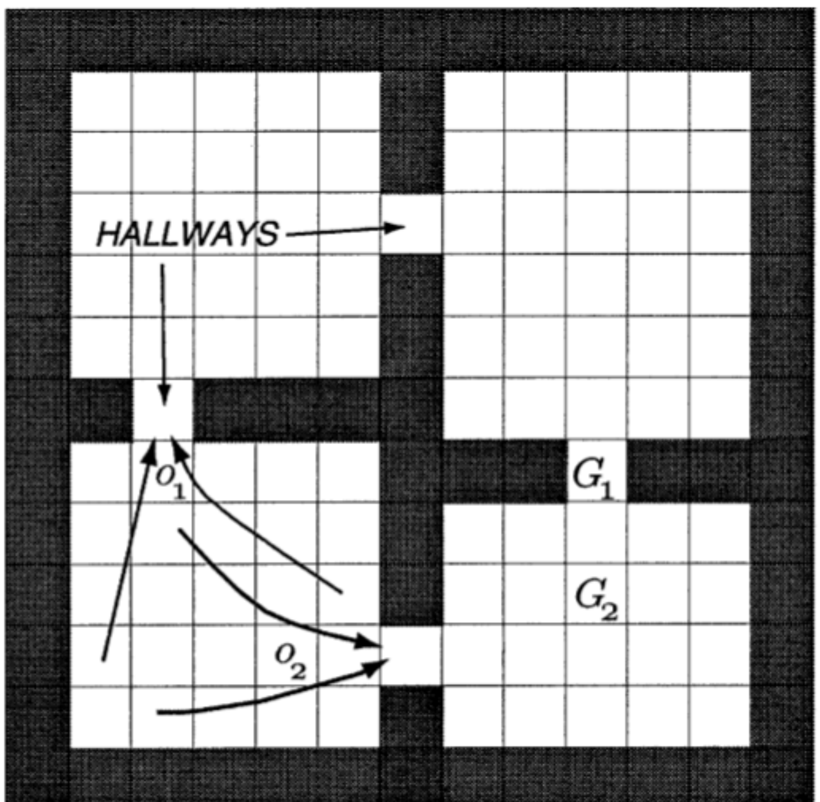
Bellman optimality Equation

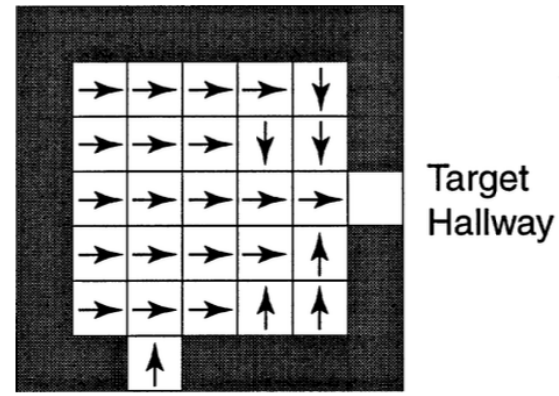
$$V_O^*(s) \stackrel{\text{def}}{=} max_{o \in O_s}[r_s^o + \sum_{s'} p(s'|s,o) V_O^*(s')]$$

$$Q_O^*(s,o) \stackrel{\text{def}}{=} r_s^o + \sum_{s'} p(s'|s,o) max_{o' \in O_{s'}} Q_O^*(s',o'),$$

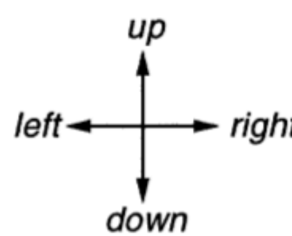
- Bellman optimality equations can be solved, exactly or approximately, using methods that generalize the usual ***DP*** and ***RL*** algorithms.

Illustration: Rooms Example





4 stochastic primitive actions

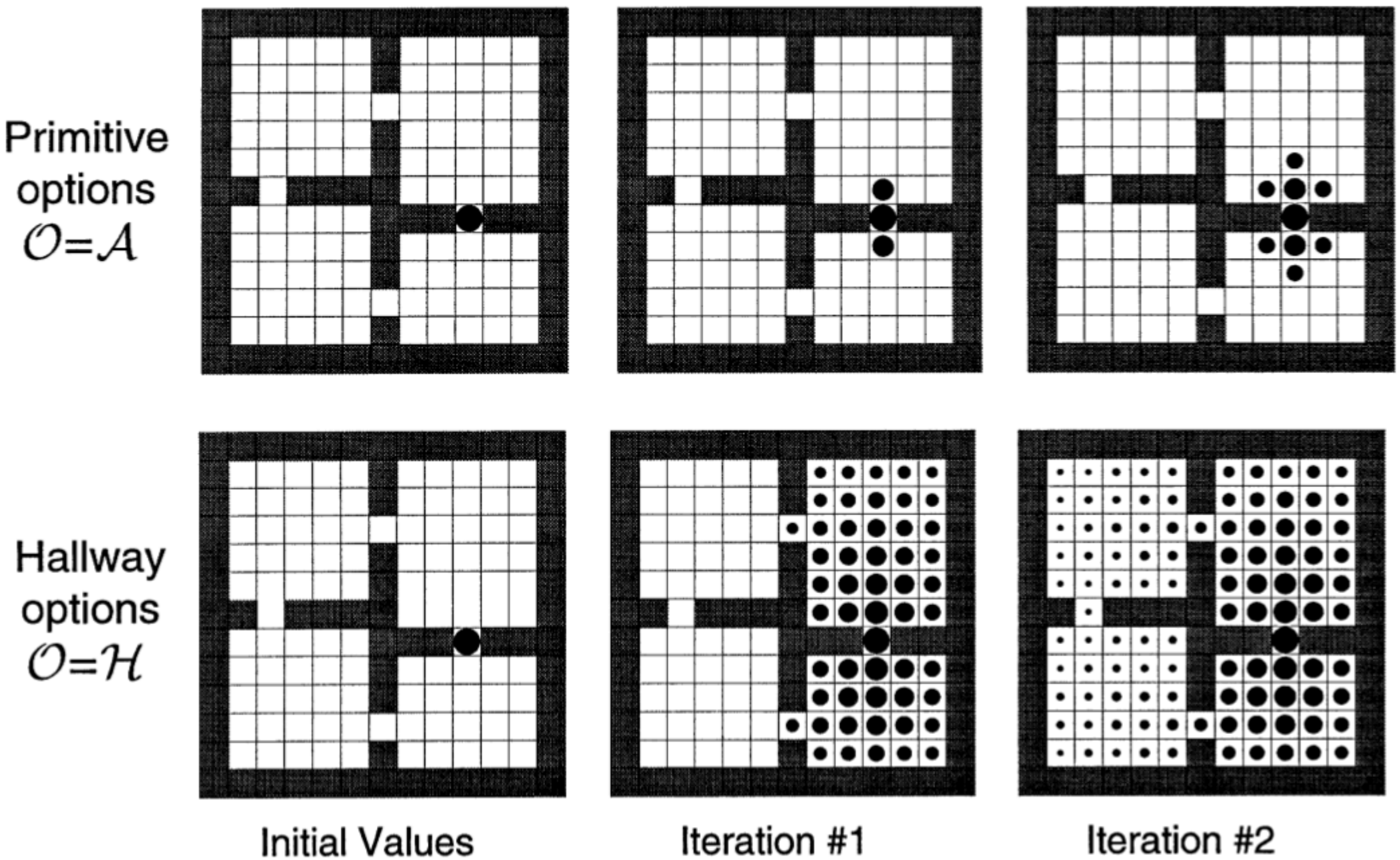


Fail 33% of the time

8 multi-step options

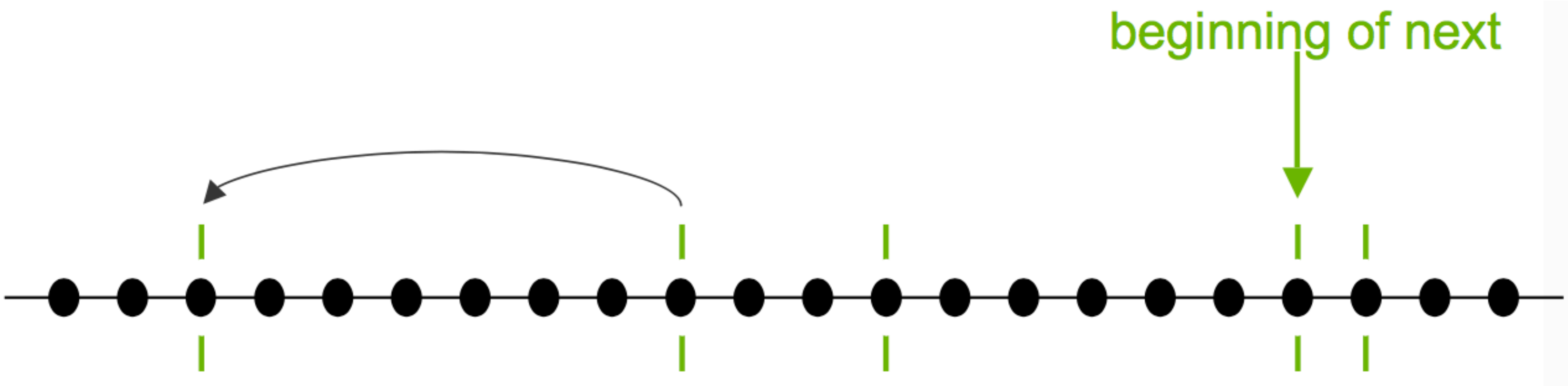
(to each room's 2 hallways)

Illustration: Rooms Example



Learning room-by-room is much faster than cell-by-cell

SMDP Q-learning backups



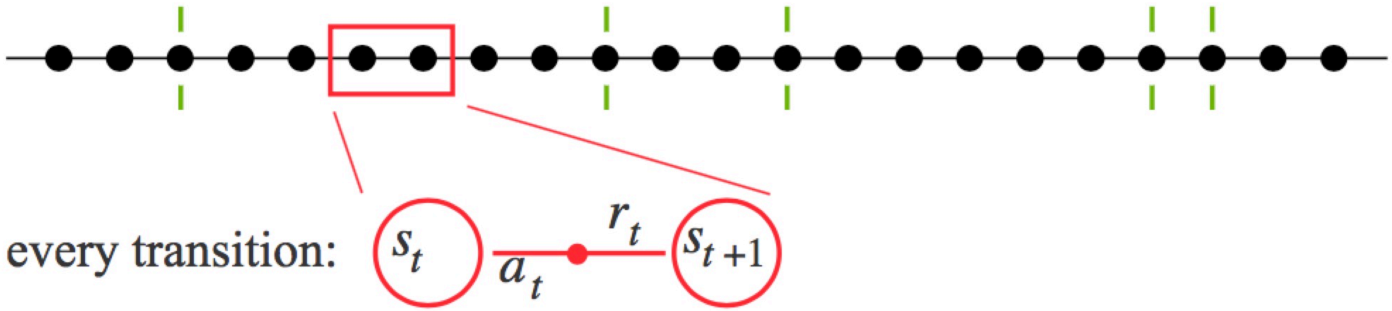
- At state s, initiate option o and execute until termination
- Observe termination state s', number of steps τ, discounted return r

$$Q_{k+1}(s,o) \stackrel{\text{def}}{=} (1-\alpha_k)Q_k(s,o) + \alpha_k(r + \gamma^{\tau} max_{o \in O_s} Q_k(s',o))$$

Looking inside options

- SMDP methods apply to options, but only when they are treated as opaque indivisible units.
- ***Interrupting*** options before they would terminate naturally according to their termination conditions.

Intra-option Q-learning



On every transition:

Update option *o every transition:*

$$Q_{k+1}(s_t,o) = (1-\alpha_k)Q_k(s_t,o) + \alpha_k\left[r_{t+1} + \gamma U_k(s_{t+1},o)\right]$$

where

$$U_k(s,o) = (1-\beta(s))Q_k(s,o) + \beta(s)\max_{o' \in O} Q_k(s,o')$$

is an estimate of the value of state-option pair (*s*,*o*) upon arrival in state *s*.

References

- D. Precup. *Temporal abstraction in reinforcement learning*. PhD thesis, University of Massachusetts Amherst, 2000.
- R. S. Sutton, D. Precup, and S. P. Singh. Between MDPs and Semi-MDPs: A framework for temporal abstraction in reinforcement learning. *Artificial Intelligence*, 112(1- 2):181–211, 1999.
- A. G. Barto and S. Mahadevan. Recent advances in hierarchical reinforcement learning. *Discrete Event Dynamic Systems*, 13(4):341 – 379, October 2003.

Options Learning

Options are typically learned using sub-goals and "pseudo-rewards".

- Tabular cases (Wiering & Schmidhuber, 1997; Schaul et al., 2015, Ofir Nachum et al. 2018) utilize row states as sub-goals.
- Pre-defined sub-goals (Tessler et al., 2016; Kulkarni et al., 2016)
- **Options Discovery**

Option Discovery Algorithms

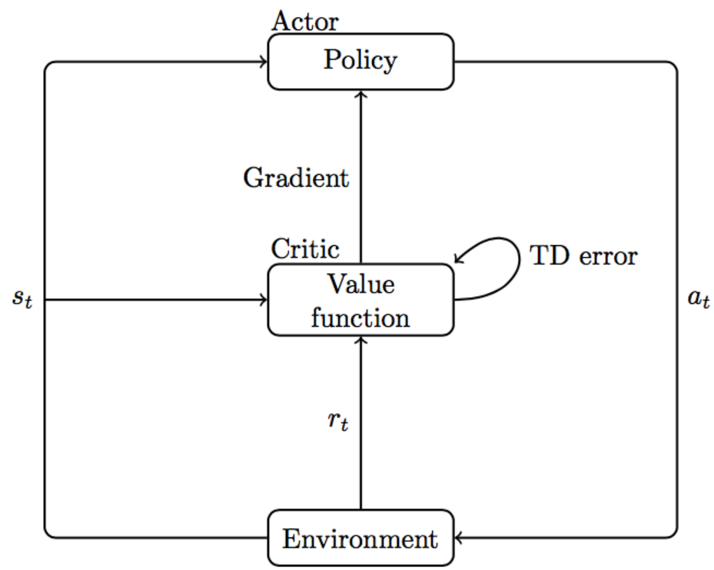
- policy gradient based methods:
  - **The Option-Critic (Bacon et al., 2017)**
  - Deep Discovery of Options (DDO) (Fox et al., 2017)
  - **FeUdal Networks (Alexander et al., 2017)**
- Information-theoretic based methods:
  - Variational Intrinsic Control (Gregor et al., 2016)
  - **Diversity is All You Need (DIAYN) (Eysenbach et al., 2018**)
  - (Florensa et al., 2017)
- Eigenoptions: (Machdo et al., 2017; Liu et al., 2017)
- Variational options discovery: VALOR (Achiam et al., 2018)

The option-critic

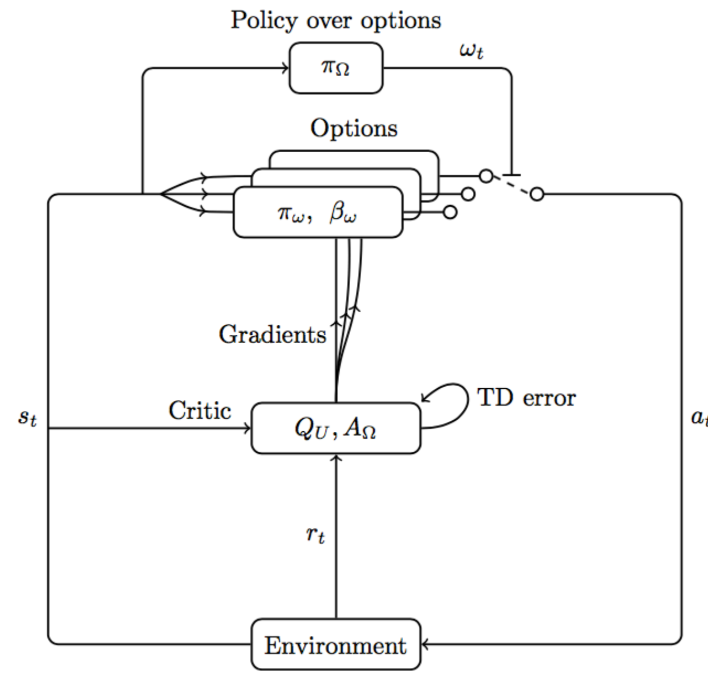
Insight

- Options can be learned end-to-end jointly with a policy-over-options using policy gradients.

Actor-Critic Architecture (Sutton 1984)



Option-Critic Architecture



- The policy (actor) is decoupled from its value function.
- The critic provides feedback to improve the actor
- Learning is fully online

- Parameterize internal policies and termination conditions
- Policy over options is computed by a separate process

The option-value function

$$Q_\Omega(s,\omega) = \sum_a \pi_{\omega,\theta}(a \mid s)\, Q_U(s,\omega,a)$$

- Where $Q_U : \mathcal{S} \times \Omega \times \mathcal{A} \to \mathbb{R}$ is the value of executing an action in the context of a state-option pair

$$Q_U(s,\omega,a) = r(s,a) + \gamma \sum_{s'} \mathrm{P}(s' \mid s,a)\, U(\omega,s')$$

$$U(\omega,s') = (1-\beta_{\omega,\vartheta}(s'))Q_\Omega(s',\omega) + \beta_{\omega,\vartheta}(s')V_\Omega(s')$$

ω – option

π_Ω – policy over options

$\pi_{\omega,\theta}$ – the intra-option policy

$\beta_{\omega,\theta}$ – termination

$U : \Omega \times \mathcal{S} \to \mathbb{R}$ – the option-value function *upon arrival,*

Main result: Gradient updates

- The **gradient wrt. the internal policy parameters** θ is given by:

$$\mathbb{E}\left[\frac{\partial \log \pi_{\omega,\theta}(a|s)}{\partial \theta} Q_U(s, \omega, a)\right]$$

  This has the usual interpretation: **take better primitives more often** inside the option

- The **gradient wrt. the termination parameters** ν is given by:

$$\mathbb{E}\left[-\frac{\partial \beta_{\omega,\nu}(s')}{\partial \nu} A_{\pi_\Omega}(s', \omega)\right]$$

  where $A_{\pi_\Omega} = Q_{\pi_\Omega} - V_{\pi_\Omega}$ is the advantage function This means that we want to **lengthen options that have a large advantage**

FeUdal Networks for Hierarchical Reinforcement Learning

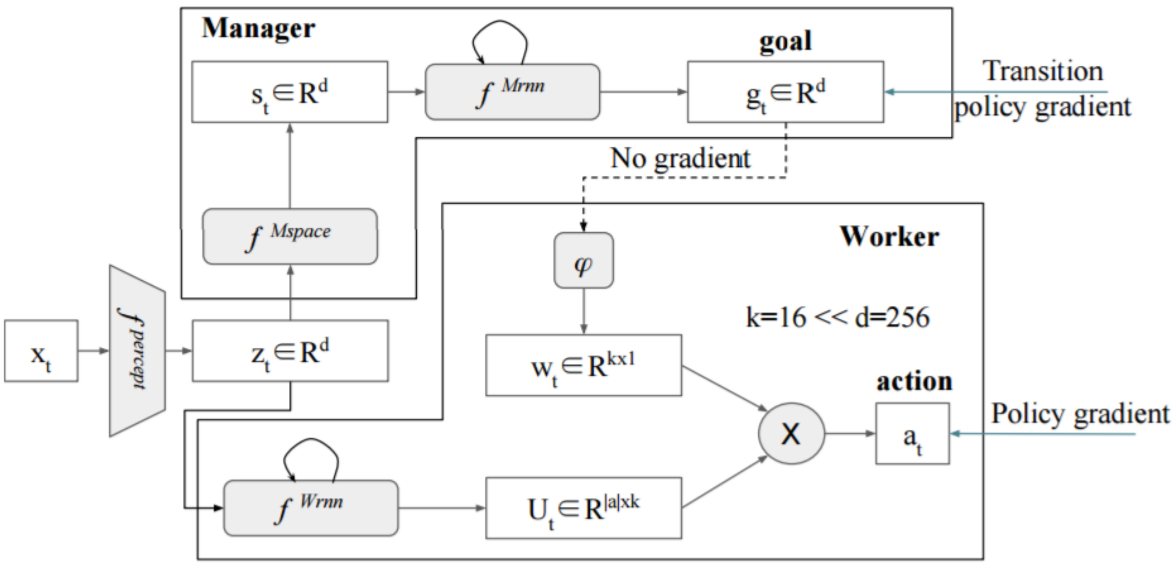
Insight

- Policy-over-options changing options at every step, i. e., high-level deviates from its own mission
- Levels of hierarchy within an agent communicate via explicit goals

FeUdal Networks (FUN, 1993)

- Proposed by Dayan & Hinton in 1993
- Let high-level managers set tasks to sub-managers, who learn how to satisfy those goals.
    - Sub-Managers learn to maximize their reinforcement in the context of the command

FuN model description



$$z_t = f^{\text{percept}}(x_t)$$

$$s_t = f^{Mspace}(z_t)$$

$$h_t^M, \hat{g}_t = f^{Mrnn}(s_t, h_{t-1}^M); g_t = \hat{g}_t / ||\hat{g}_t||;$$

$$w_t = \phi(\sum_{i=t-c}^{t} g_i)$$

$$h^W, U_t = f^{Wrnn}(z_t, h_{t-1}^W)$$

$$\pi_t = SoftMax(U_t w_t)$$

z: Embedding of env. x

h^M: Internal state of manager

h^W: Internal state of worker

g: Goal

w: Embedding of goal g

c: Prediction horizon

U: Output of worker

π: Vector of prob. over actions

$f^{percept}$: CNN, 1st layer: 16 8x8 filters w/ stride 4, 2nd layer 32 4x4 filters w/ stride 2, fully connected layer has 256 hidden units.

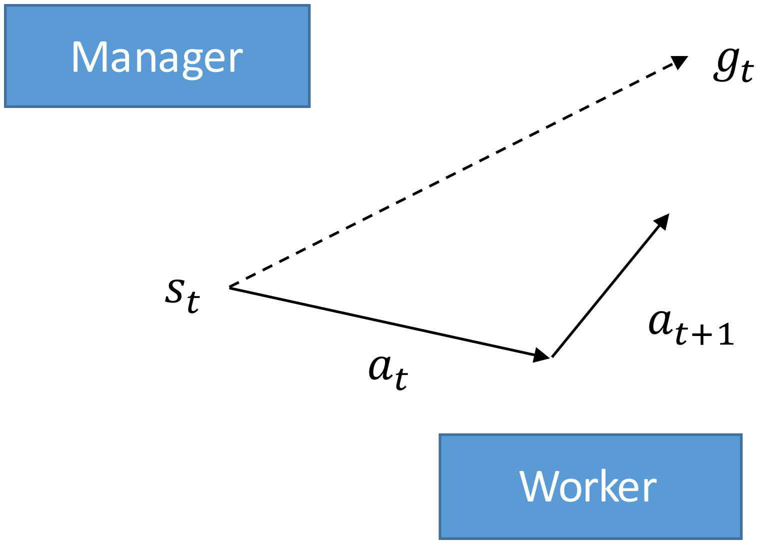
f^{Mspace}: Fully conn. layer, computes state space.

f^{Wrnn}: Standard LSTM w/ 256 hidden units, computes goal.

f^{Mrnn}: Dilated LSTM w/ 256 hidden units (will be explained detailed later).

FuN model description

$s_t = \phi(x_t) \in R^d$



- Complementary representations
- Multiple time scale

Learning

Bad idea:

- train feudal network end-to-end using a policy gradient algorithm operating on the actions taken by the Worker

Good idea:

- independently train Manager to predict advantageous directions in state space and to intrinsically reward the Worker to follow these directions

The agents goal

Maximize the discounted return

$$R_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1}$$

- The agent's behavior is defined by its action-selection policy π. FuN produces a distribution over possible actions.

Manager's transition policy

- Consider $g_t = g(s_t, \theta)$

$\mu(.)$: High-level policy selecting among subpolicies

o: Sub-policy

- Transition policy:

$\pi^{TP}(s_{t+c}|s_t) = p(s_{t+c}|s_t, \mu(s_t, \theta))$

- Transition policy gradient:

$\nabla_\theta {\pi_t}^{TP} = \mathbb{E}\big[\big(R_t - V(s_t)\big)\nabla_\theta \log p(s_{t+c}|s_t, \mu(s_t, \theta))\big]$

- It is assumed that,

$p(s_{t+c}|s_t, \mu(s_t, \theta)) \propto e^{d_{cos}(s_{t+c} - s_t, g_t)}$ (von Mises-Fisher distribution)

Gradient for the goal:

$$\nabla g_t = A_t{}^M \nabla_\theta d_{cos}(s_{t+c} - s_t, g_t(\theta))$$

where

$A_t^M \;=\; R_t - V_t^M(x_t, \theta)$ – advantage function

$V_t^M(x_t, \theta)$ – value function estimate from the internal critic

$d_{\cos}(\alpha, \beta) \;=\; \alpha^T \beta / (|\alpha||\beta|)$ – cosine similarity

Workers intrinsic reward

- Worker's policy π is trained to maximize $R_t + \alpha {R_t}^I$.

${r_t}^I = \frac{1}{c}\sum_{i=1}^{c} d_{cos}(s_t - s_{t-i}, g_{t-i})$: Intrinsic reward

R_t: Extrinsic discounted return

α: Hyperparameter to blend intinsic and extrinsic reward

${R_t}^I$: Intrinsic discounted return

c: horizon

- Worker policy gradient:

$$\nabla \pi_t = {A_t}^D \nabla_\theta \log \pi(a_t | x_t; \theta)$$

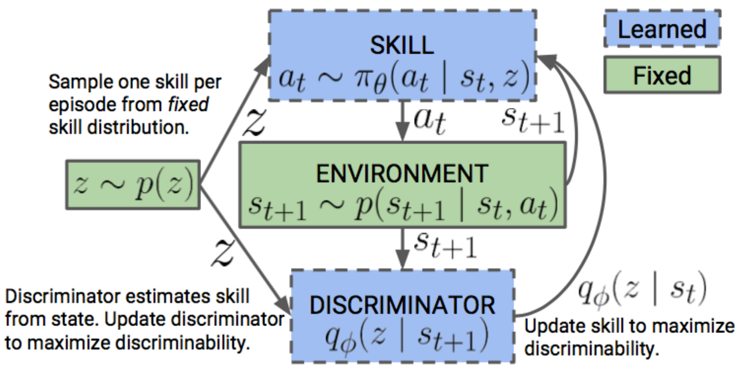
$${A_t}^D = (R_t + \alpha {R_t}^I - {V_t}^M(x_t, \theta))$$

Diversity is All You Need: Learning Skills without a Reward Function

Insight

- Learning skills without reward

FuN



Algorithm 1: DIAYN

while *not converged* **do**
  Sample skill $z \sim p(z)$ and initial state $s_0 \sim p_0(s)$
  for $t \leftarrow 1$ **to** *steps_per_episode* **do**
    Sample action $a_t \sim \pi_\theta(a_t \mid s_t, z)$ from skill.
    Step environment: $s_{t+1} \sim p(s_{t+1} \mid s_t, a_t)$.
    Compute $q_\phi(z \mid s_{t+1})$ with discriminator.
    Set skill reward $r_t = \log q_\phi(z \mid s_{t+1}) - \log p(z)$
    Update policy (θ) to maximize r_t with SAC.
    Update discriminator (ϕ) with SGD.

Z ~ p(z) – a latent variable; A policy conditioned on a fixed Z as a “skill”

How it works

$$
\begin{aligned}
\mathcal{F}(\theta) &\triangleq I(S; Z) + \mathcal{H}[A \mid S] - I(A; Z \mid S) \\
&= (\mathcal{H}[Z] - \mathcal{H}[Z \mid S]) + \mathcal{H}[A \mid S] - (\mathcal{H}[A \mid S] - \mathcal{H}[A \mid S, Z]) \\
&= \mathcal{H}[Z] - \mathcal{H}[Z \mid S] + \mathcal{H}[A \mid S, Z]
\end{aligned}
$$

$I\ (\cdot;\cdot)$ and $H[\cdot]$ – mutual information and Shannon entropy
Maximize $I\ (S;\ Z\ )$ –the skill should control which states the agent visits; the skill can be inferred from the states visited.
Minimize $I(A;\ Z \mid S)$ – that states, not actions, are used to distinguish skills
Maximize $H[A \mid S]$ – maximize the entropy of mixture policy

Implementation

$$\begin{aligned}\mathcal{F}(\theta) &\triangleq I(S;Z) + \mathcal{H}[A \mid S] - I(A;Z \mid S) \\ &= (\mathcal{H}[Z] - \mathcal{H}[Z \mid S]) + \mathcal{H}[A \mid S] - (\mathcal{H}[A \mid S] - \mathcal{H}[A \mid S, Z]) \\ &= \mathcal{H}[Z] - \mathcal{H}[Z \mid S] + \mathcal{H}[A \mid S, Z]\end{aligned}$$

Encourage $p(z)$ to have high entropy – Fix $p(z)$ to be uniform
Minimize $H[Z| S]$ – Add in to the reward function . $r_z(s,a) \triangleq \log q_\phi(z \mid s) - \log p(z)$
Maximize $H[A| S, Z]$ – using soft actor critic.

As we cannot integrate over all states and skills to compute $p(z \mid s)$ exactly, we approximate this posterior with a learned discriminator $q\varphi(z \mid s)$.

Review

- Options
  - A generalization of actions
- SMDP
  - MDP + Options = SMDP
  - Temporal abstraction
- Options discovery
  - The option-critic
  - FeUdal
  - DIAYN

Thanks